647510 **BSCgraCC6010** **Seat No :________**

B.Sc. Semester - 6 (*CBCS*) Examination
May/June-2021 (OLD COURSE)
GRAPH THEORY AND COMPLEX ANALYSIS -2(CORE)

Time: 1:30 Hours **Marks: 42**

Instructions:

1. Figures to the right indicate marks.
2. There are five questions in the question paper.
3. Answer any three of the following questions.

Q.1(A). Answer any one out of two. [7]

(1) Define Euler graph. State and Prove necessary and sufficient condition for being Euler graph.

(2) Prove that in complete graph with n vertices there are $\frac{n-1}{2}$ edge disjoint Hamiltonian circuits, where n is odd no. ≥ 3.

Q.1(B). Answer any one out of two. [4]

(1) Prove that if a graph has exactly two vertices of odd degree, there must be a path joining these two vertices.

(2) Prove that a tree T with n vertices has n-1 edges.

Q.1(C). Answer any one out of two. [3]

(1) Define: Parallel edges, Complete graph and Euler graph.

(2) Prove that in complete graph number of edges is $\frac{n(n-1)}{2}$.

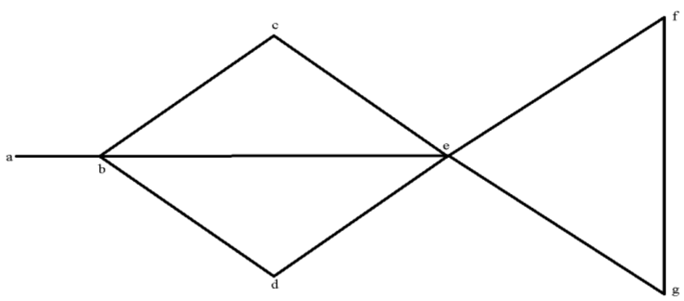
Q.2 (A). Answer any one out of two. [7]

(1). State and prove Euler's formula for a connected planer graph.

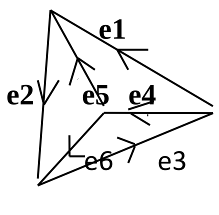
(2). In usual notation for any simple, connected planer graph prove that $e \geq \frac{3}{2}f$ and $e \leq 3n-6$.

Q.2 (B). Answer any one out of two. [4]

(1). Find maximal independent set for following graph.



(2). Find minimal decyclization of the following graph.



Q.2(C). Answer any one out of two. [3]

(1). Define Proper colouring of graph, Chromatic number.

(2). Define Path matrix with example.

Q.3(A). Answer any one out of two. [7]

(1). In usual notation prove that the bilinear transformation maps three given distinct points z_1,z_2 and z_3 onto three specified distinct points w_1,w_2 and w_3 is $\frac{(w-w_1)(w_2-w_3)}{(w-w_3)(w_2-w_1)} = \frac{(z-z_1)(z_2-z_3)}{(z-z_3)(z_2-z_1)}$.

(2) Discuss the transformation W= sin z.

Q.3(B). Answer any one out of two. [4]

(1). Obtain the transformation of the region x ≥ C_1 of z - plane under the mapping $\frac{1}{z}$.

(2). Find the mobious mapping which transforms points z = -1, ∞, 1 of z-plane in to w = 2, 1, 0 of w-plane.

Q.3(C). Answer any one out of two. [3]

(1). Prove that the transformation w = $\frac{2z+3}{z-4}$ maps the circle $x^2 + y^2 - 4x = 0$ of the z-plane into a line 4u+3=0 in w-plane.

(2). Find the fixed point of transformation w = $\frac{z-1}{z+1}$.

Q.4(A). Answer any one out of two. [7]

(1). State and prove Taylor's theorem for an analytic function.

(2). State and prove Laurent's theorem for an analytic function.

Q.4(B). Answer any one out of two. [4]

(1). Expand $\frac{1}{z}$ in increasing power of (z-1).

(2).Expand $\frac{z}{(z-1)(z-3)}$ in Laurent's series for 0 < | z-1 | < 2.

Q.4(C). Answer any one out of two. [3]

(1). Find the region of convergence and radius of convergence for $\sum_1^\infty \frac{z^n}{2^n+1}$.

(2). Prove: coshz = $\sum_{n=0}^\infty \frac{z^{2n}}{(2n)!}$.

Q.5(A). Answer any one out of two. [7]

(1). Prove by using Residue Theorem $\int_0^\infty \frac{dx}{(x^2+a^2)^2} = \frac{\pi}{4a^3}$; a > 0.

(2). Prove by using Residue Theorem $\int_0^\pi \frac{d\theta}{(2+cos\theta)^2} = \frac{2\pi}{3\sqrt{3}}$.

Q.5(B). Answer any one out of two. [4]

(1). State and prove Cauchy's residue theorem.

(2).Evaluate $\int_c \frac{5z-2}{(z-1)}$ dz; C: |z| =2.

Q.5(C). Answer any one out of two. [3]

(1). Discuss the method for finding the residue of f(z) at the simple pole z_0

(2). Find: Res$\left(\frac{e^{2z}}{(z-1)^2}, 1\right)$

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